

Mathematics Progression in Calculation (EYFS, Y1 and Y2)

INTRODUCTION

This document has been created to set out the progression of skills in mathematical calculations taught at Shalford Infant and Nursery School. The document shows the progression in two directions: horizontal progress is shown through the transition from use of concrete apparatus to pictorial diagrams, before moving in to abstract representations of mathematical learning. Vertical progress is charted in terms of matching taught strategies and skills to the requirements of the National Curriculum in the different year groups.

Concrete, pictorial and Abstract (CPA) approach

We recognise that the **Concrete Pictorial Abstract (CPA)** approach is highly effective in the teaching of Maths to develop conceptual understanding. This approach will vary between year groups and the individual abilities of children within each class.

Manipulatives (objects), pictorial representations, words, numbers and symbols are everywhere. The mastery approach incorporates all of these to help children explore and demonstrate mathematical ideas, enrich their learning experience and deepen understanding. Together, these elements help cement knowledge so pupils truly understand what they've learnt.

All pupils, when introduced to a key new concept, should have the opportunity to build competency in this topic by taking this approach. Pupils are encouraged to physically represent mathematical concepts. Objects (manipulatives) and pictures are used to demonstrate and visualise abstract ideas, alongside numbers and symbols.

Concrete - The doing stage

There is a clear focus on the use of manipulatives and visual images to support understanding in every year group. Each new concept or calculation strategy will be introduced using appropriate manipulatives.

Pictorial - The seeing stage

A child has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or a picture of the problem.

Abstract- The symbolic stage

A child is now capable of representing problems by using mathematical notation, for example $10 \div 2 = 5$

EARLY YEARS/ KEY STAGE 1

Children develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.

Key language: whole, part, ones, ten, tens, number bond, add, addition, plus, total, altogether, subtract, subtraction, find the difference, take away, minus, less, more, group, share, equal, equals, is equal to, groups, equal groups, times, multiply, multiplied by, divide, share, shared equally, times-table

Addition and subtraction: Children first learn to connect addition and subtraction with counting, but they soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations.

A key idea is that children will select methods and approaches based on their number sense. For example, in Year 1, when faced with 15 – 3 and 15 – 13, they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods.

In Year 2, they will start to see calculations presented in a column format, although this is not expected to be formalised until KS2. We show the column method in Year 2 as an option; teachers may not wish to include it until Year 3.

Multiplication and division: Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division.

They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation. In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations. Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting.

Fractions: In Year 1, children encounter halves and quarters, and link this with their understanding of sharing. They experience key spatial representations of these fractions, and learn to recognise examples and non-examples, based on their awareness of equal parts of a whole.

In Year 2, they develop an awareness of unit fractions and experience non-unit fractions, and they learn to write them and read them in the common format of numerator and denominator.

Calculation Policy - EYFS (Nursery & Reception)				
Addition	Subtraction	Multiplication	Division	
Children are encouraged to gain a sense of the number system through the use of counting concrete objects.	Children are encouraged to gain a sense of the number system through the use of counting concrete objects.	Children use concrete objects to make and count equal groups of objects.	Children use concrete objects to count and share equally into 2 groups.	
or counting concrete objects.	of counting concrete objects.	They will count on in twos using a bead	6 cakes shared between 2 people each person gets 3 cakes. 6 ÷2 = 3	
They combine objects in practical ways and count all.	They understand subtraction as counting out.	string and number line. They understand doubling as repeated		
They understand addition as counting	They begin to count back in ones and	addition.	They count a set of objects and halve	
on and will	twos using objects, cubes, bead strings	2 + 2 = 4	them by making two equal groups.	
count on in	and number lines.			
ones and 1 2 3 4 5 6 7 8 9 10	•••••	They use concrete	They understand sharing and halving as	
twos using		and pictorial representation to	dividing by 2.	
cubes, OOOOOO	1 2 3 4 5 6 7 8 9 10	record their	They will begin to use objects to make	
objects, bead strings and number lines.	They use concrete and pictorial representation to record their	calculations. Higher attaining children may be	groups of 2 from a given amount.	
They use concrete and pictorial	calculations.	able to represent their calculations using	They use concrete and pictorial	
representation to record their		symbols and numbers within a written calculation.	representation to record their	
calculations.	They begin to use - and =	calculation.	calculations.	
They begin to use + and = They are encouraged to	They are encouraged to develop a			
develop a mental picture of	mental picture of the number system in			
the number system in their	their heads to use for calculations.		Higher attaining children may be able to	
heads to use for calculations. *********	Higher attaining children may be able to		represent their calculations using symbols and numbers within a written	
Higher attaining children may be able to represent their calculations using symbols and numbers within a written	represent their calculations using symbols and numbers within a written calculation.		calculation.	
calculation.				

The statutory requirements related to number for children at the expected level of development at the end of the EYFS are as follows:

Statutory ELG: Number

Children at the expected level of development will:

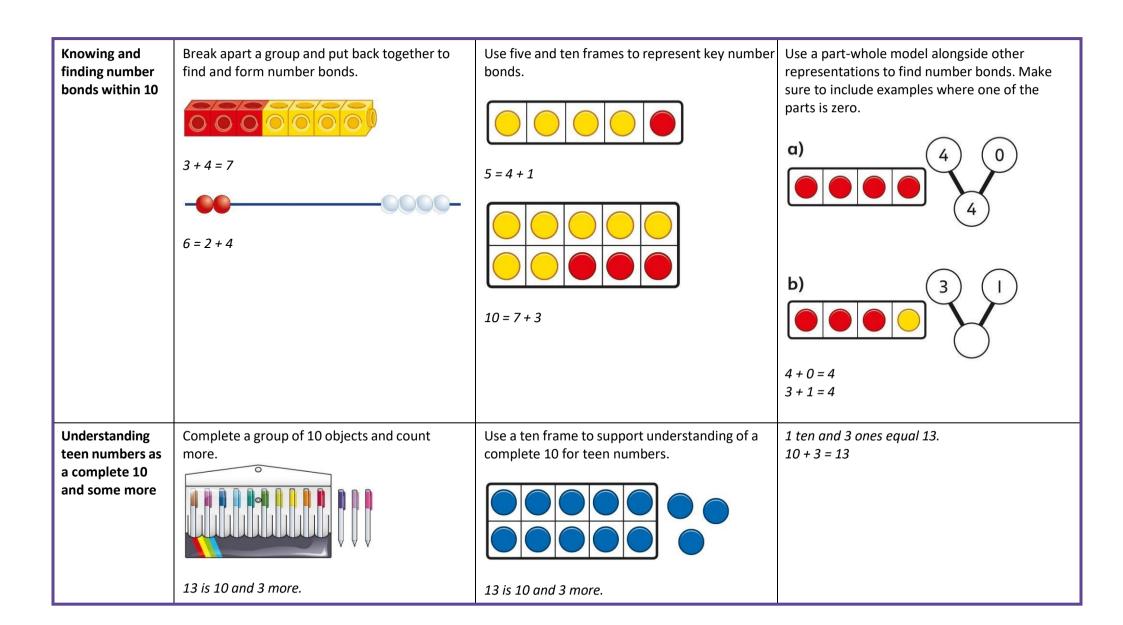
- Have a deep understanding of number to 10, including the composition of each number;- Subitise (recognise quantities without counting) up to 5;
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Statutory ELG: Numerical Patterns

Children at the expected level of development will:

- Verbally count beyond 20, recognising the pattern of the counting system;
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity:
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

	Year 1			
Year 1 Addition	Concrete	Pictorial	Abstract	
Counting and adding more	Children add one more person or object to a group to find one more.	Children add one more cube or counter to a group to represent one more.	Use a number line to understand how to link counting on with finding one more.	
		One more than 4 is 5.	One more than 6 is 7. 7 is one more than 6.	
			Learn to link counting on with adding more than one. 1	
Understanding part-part-whole relationship	Sort people and objects into parts and understand the relationship with the whole.	Children draw to represent the parts and understand the relationship with the whole. The parts are 1 and 5. The whole is 6.	Use a part-whole model to represent the numbers. 6 4 6 + 4 = 10	
	The parts are 2 and 4. The whole is 6.	,	6 + 4 = 10	



Adding by counting on	Children use knowledge of counting to 20 to find a total by counting on using people or objects.	Children use counters to support and represent their counting on strategy.	Children use number lines or number tracks to support their counting on strategy.
	8 on the bus	7 on the bus	7 7 + 5 =
Adding the 1s	Children use bead strings to recognise how to add the 1s to find the total efficiently. 2 + 3 = 5 12 + 3 = 15	Children represent calculations using ten frames to add a teen and 1s.	Children recognise that a teen is made from a 10 and some 1s and use their knowledge of addition within 10 to work efficiently. 3 + 5 = 8 So, 13 + 5 = 18
		2 + 3 = 5 12 + 3 = 15	
Understanding 10s and 1s	Group objects into 10s and 1s. Bundle straws to understand unitising of 10s.	Understand 10s and 1s equipment, and link with visual representations on ten frames.	Represent numbers on a place value grid, using equipment or numerals. Tens Ones 3 2 Tens Ones 4 3

Year 1 Subtraction	Concrete	Pictorial	Abstract
Counting back and taking away	Children arrange objects and remove to find how many are left. 1 less than 6 is 5. 6 subtract 1 is 5.	Children draw and cross out or use counters to represent objects from a problem. 9 - = There are children left.	Children count back to take away and use a number line or number track to support the method. 876 $9-3=6$
Finding a missing part, given a whole and a part	Children separate a whole into parts and understand how one part can be found by subtraction. 8 - 5 = ?	Children represent a whole and a part and understand how to find the missing part by subtraction. 5 - 4 =	Children use a part-whole model to support the subtraction to find a missing part. 7 - 3 = ? Children develop an understanding of the relationship between addition and subtraction facts in a part-whole model. - = - = - = - = - = - = - = - = - = -

Finding the difference	Arrange two groups so that the difference between the groups can be worked out.	Represent objects using sketches or counters to support finding the difference.	Children understand 'find the difference' as subtraction.
	8 is 2 more than 6. 6 is 2 less than 8. The difference between 8 and 6 is 2.	5 - 4 = 1 The difference between 5 and 4 is 1.	0 2 3 4 5 6 7 8 9 10 10 - 4 = 6 The difference between 10 and 6 is 4.
Subtraction within 20	Understand when and how to subtract 1s efficiently. Use a bead string to subtract 1s efficiently. $5-3=2$ $15-3=12$	Understand when and how to subtract 1s efficiently. Output Description: $5-3=2$ $15-3=12$	Understand how to use knowledge of bonds within 10 to subtract efficiently. $5-3=2$ $15-3=12$
Subtracting 10s and 1s	For example: 18 – 12 Subtract 12 by first subtracting the 10, then the remaining 2. First subtract the 10, then take away 2.	For example: 18 – 12 Use ten frames to represent the efficient method of subtracting 12. First subtract the 10, then subtract 2.	Use a part-whole model to support the calculation. 10 4 19 - 14 19 - 10 = 9 9 - 4 = 5 So, 19 - 14 = 5

Subtraction For example: 12 – 7 Represent the use of bonds using ten frames. Use a number line and a part-whole model to bridging 10 using support the method. Arrange objects into a 10 and some 1s, then number bonds decide on how to split the 7 into parts. 13 – 5 For 13 – 5, I take away 3 to make 10, then take away 2 to make 8. 7 is 2 and 5, so I take away the 2 and q then the 5. Year 1 Concrete **Pictorial** Abstract Multiplication **Describe equal** Children arrange objects in equal and unequal Children draw and represent equal and Three equal groups of 4. groups using groups and understand how to recognise unequal groups. Four equal groups of 3. whether they are equal. words Finding the total 100 squares and ten frames support counting Use a number line to support repeated addition of equal groups in 2s, 5s and 10s. through counting in 2s, 5s and 10s. by counting in 2s, 5s and 10s 10 10 There are 5 pens in each pack ... 5...10...15...20...25...30...35...40... 21 22 23 24 25 26 27 28 29 30 20 30 40 50 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50

Year 1 Division	Concrete	Pictorial	Abstract
Grouping	Learn to make equal groups from a whole and find how many equal groups of a certain size can be made.	Represent a whole and work out how many equal groups.	Children may relate this to counting back in steps of 2, 5 or 10.
	Sort a whole set people and objects into equal groups.	000000000	
		There are 10 in total. There are 5 in each group. There are 2 groups.	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
	There are 10 children altogether. There are 2 in each group. There are 5 groups.		
Sharing	Share a set of objects into equal parts and work out how many are in each part.	Sketch or draw to represent sharing into equal parts. This may be related to fractions.	10 shared into 2 equal groups gives 5 in each group.

	Year 2			
Year 2 Addition	Concrete	Pictorial	Abstract	
Bridging	Bridging the 10 using number bonds Children use a bead string to complete a 10 and understand how this relates to the addition. 7 add 3 makes 10. So, 7 add 5 is 10 and 2 more.	Bridging the 10 using number bonds Children use counters to complete a ten frame and understand how they can add using knowledge of number bonds to 10.	Bridging the 10 using number bonds Use a part-whole model and a number line to support the calculation. 4 9 10 11 12 13 9+4=13	
Adding 10s	Use known bonds and unitising to add 10s. I know that 4 + 3 = 7. So, I know that 4 tens add 3 tens is 7 tens.	Use known bonds and unitising to add 10s. ###################################	Use known bonds and unitising to add 10s. 4 + 3 = 4 + 3 = 7 4 tens + 3 tens = 7 tens 40 + 30 = 70	

Adding a 1-digit number to a 2-digit number not bridging a 10 Add the 1s to find the total. Use known bonds within 10.





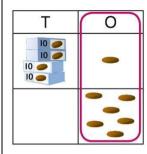






41 is 4 tens and 1 one. 41 add 6 ones is 4 tens and 7 ones.

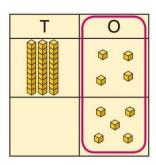
This can also be done in a place value grid.



Add the 1s.

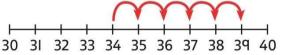


34 is 3 tens and 4 ones.
4 ones and 5 ones are 9 ones.
The total is 3 tens and 9 ones.



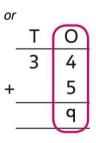
Add the 1s.

Understand the link between counting on and using known number facts. Children should be encouraged to use known number bonds to improve efficiency and accuracy.



This can be represented horizontally or vertically.

$$34 + 5 = 39$$



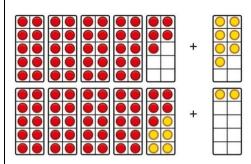
Adding a 1-digit number to a 2-digit number bridging 10 Complete a 10 using number bonds.



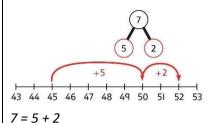


There are 4 tens and 5 ones.
I need to add 7. I will use 5 to complete a 10, then add 2 more.

Complete a 10 using number bonds.



Complete a 10 using number bonds.

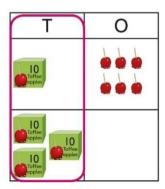


7 = 5 + 245 + 5 + 2 = 52

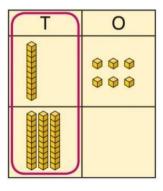
Adding a Exchange 10 ones for 1 ten. Exchange 10 ones for 1 ten. Exchange 10 ones for 1 ten. 1-digit number to a 2-digit number 0 2 4 using exchange 9999 9999 9999 0 0 99 Adding a Add the 10s and then recombine. Add the 10s and then recombine. Add the 10s and then recombine. multiple of 10 to a 2-digit number *37 + 20 = ?* 30 + 20 = 5050 + 7 = 57999999 27 is 2 tens and 7 ones. 37 + 20 = 5750 is 5 tens. 66 is 6 tens and 6 ones. 66 + 10 = 76There are 7 tens in total and 7 ones. So, 27 + 50 is 7 tens and 7 ones. A 100 square can support this understanding. 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 3 4 42 43 44 45 46 47 48 49 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

Adding a
multiple of 10 to
a 2-digit number
using columns

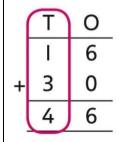
Add the 10s using a place value grid to support.



16 is 1 ten and 6 ones. 30 is 3 tens. There are 4 tens and 6 ones in total. Add the 10s using a place value grid to support.



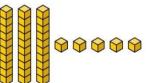
16 is 1 ten and 6 ones. 30 is 3 tens. There are 4 tens and 6 ones in total. Add the 10s represented vertically. Children must understand how the method relates to unitising of 10s and place value.



1 + 3 = 4 1 ten + 3 tens = 4 tens 16 + 30 = 46

Adding two 2-digit numbers

Add the 10s and 1s separately.

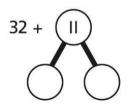


5 + 3 = 8There are 8 ones in total.

3 + 2 = 5There are 5 tens in total.

35 + 23 = 58

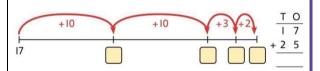
Add the 10s and 1s separately. Use a part-whole model to support.



11 = 10 + 1 32 + 10 = 4242 + 1 = 43

32 + 11 = 43

Add the 10s and the 1s separately, bridging 10s where required. A number line can support the calculations.



17 + 25

Adding two 2-digit numbers	Add the 1s. Then add the 10s.	Add the 1s. Then add the 10s.
using a place value grid	Tens Ones	T O 3 2 + 1 4 6
	Tens Ones	T O 3 2 1 4 4 6
Adding two	Add the 1s. Exchange 10 ones for a ten. Then add	Add the 1s. Exchange 10 ones for a ten. Then
2-digit numbers	the 10s.	add the 10s.
with exchange	Tens Ones 3 6 4 2 9 Tens Ones 4 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9	T O 3 6 + 2 q 5 5 T O 3 6 + 2 q 6 5
	Tens Ones	

Year 2 Subtraction	Concrete	Pictorial	Abstract
Subtracting multiples of 10	Use known number bonds and unitising to subtract multiples of 10.	Use known number bonds and unitising to subtract multiples of 10.	Use known number bonds and unitising to subtract multiples of 10.
		100	7 70 2 5 20 50
	8 subtract 6 is 2. So, 8 tens subtract 6 tens is 2 tens.	10 – 3 = 7 So, 10 tens subtract 3 tens is 7 tens.	7 tens subtract 5 tens is 2 tens. 70 – 50 = 20
Subtracting a single-digit number	Subtract the 1s. This may be done in or out of a place value grid.	Subtract the 1s. This may be done in or out of a place value grid.	Subtract the 1s. Understand the link between counting back and subtracting the 1s using known bonds.
	T O		30 31 32 33 34 35 36 37 38 39 40 T O 3 9 - 3
Subtracting a single-digit number bridging 10	Bridge 10 by using known bonds.	Bridge 10 by using known bonds.	Bridge 10 by using known bonds. -4 -4 -6 16 17 18 19 20 21 22 23 24 25 26
	35 – 6 I took away 5 counters, then 1 more.	35 – 6 First, I will subtract 5, then 1.	24 - 6 = ? 24 - 4 - 2 = ?

Subtracting a Exchange 1 ten for 10 ones. Exchange 1 ten for 10 ones. Exchange 1 ten for 10 ones. This may be done single-digit in or out of a place value grid. number using 0 exchange 0 000 9 9 10 10 Т 0 0 000 10 888 *25 - 7 = 18* Subtracting a Subtract by taking away. Subtract the 10s and the 1s. Subtract the 10s and the 1s. 2-digit number 000000000 This can be represented on a 100 square. This can be represented on a number line. 12 13 14 15 16 17 18 19 20 53 63 64 21 22 23 24 25 26 27 28 29 30 64 - 41 = ?31 32 33 34 35 36 37 38 39 40 42 43 44 45 46 47 48 49 50 64 - 1 = 6352 53 54 55 56 57 58 59 60 63 - 40 = 2361 62 63 64 65 66 67 68 69 70 64 - 41 = 2371 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 61 - 18 I took away 1 ten and 8 ones. 91 92 93 94 95 96 97 98 99 100 - 10 26 46 - 20 = 26 26 - 5 = 2146 - 25 = 21

Subtracting a 2-digit number using place value and columns	Subtract the 1s. Then subtract the 10s. This may be done in or out of a place value grid. T O O O O O O O O O O O O O O O O O O	Subtract the 1s. Then subtract the 10s. Tens Ones	Using column subtraction, subtract the 1s. Then subtract the 10s. TO 45 -12 3 TO 45 -12 33 3
Subtracting a 2-digit number with exchange		Exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s. Tens Ones Tens Ones Tens Ones Tens Ones Tens Ones Tens Ones	Using column subtraction, exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s. $ \frac{T}{4} \frac{O}{5} $ $ -2 7 $ $ \frac{T}{4} \frac{O}{5} $ $ -2 7 $ $ \frac{T}{4} \frac{O}{5} $ $ -2 7 $ $ \frac{T}{4} \frac{O}{5} $ $ \frac{7}{4} \frac{O}{5} $ $ \frac{3}{4} \frac{O}{5} $ $ \frac{7}{4} \frac{O}{5} $

Year 2 Multiplication	Concrete	Pictorial	Abstract
Equal groups and repeated addition	Recognise equal groups and write as repeated addition and as multiplication. 3 groups of 5 chairs 15 chairs altogether	Recognise equal groups using standard objects such as counters and write as repeated addition and multiplication. 3 groups of 5	Use a number line and write as repeated addition and as multiplication. $ \begin{array}{cccccccccccccccccccccccccccccccccc$
Using arrays to represent multiplication and support understanding	Understand the relationship between arrays, multiplication and repeated addition.	Understand the relationship between arrays, multiplication and repeated addition. 4 groups of 5 5 groups of 5	Understand the relationship between arrays, multiplication and repeated addition. $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
Understanding commutativity	Use arrays to visualise commutativity. I can see 6 groups of 3. I can see 3 groups of 6.	Form arrays using counters to visualise commutativity. Rotate the array to show that orientation does not change the multiplication. This is 2 groups of 6 and also 6 groups of 2.	Use arrays to visualise commutativity. $4+4+4+4+4=20$ $5+5+5=20$ $4 \times 5 = 20$ and $5 \times 4 = 20$

Learning ×2, ×5 and ×10 table facts

Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts.







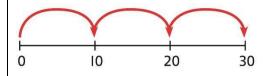
3 groups of 10 ... 10, 20, 30 $3 \times 10 = 30$

Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.





000000000



$$10 + 10 + 10 = 30$$

 $3 \times 10 = 30$

Understand how the times-tables increase and contain patterns.



10 10

10 10 10

10 10 10 10

10 10 10 10

10 10 10 10 10

10 10 10 10 10 10

10 10 10 10 10 10 10

10 10 10 10 10 10 10 10

10 10 10 10 10 10 10 10 10

10 10 10 10 10 10 10 10 10 10

10 10 10 10 10 10 10 10 10 10 10

 $5 \times 10 = 50$

 $6 \times 10 = 60$

Year 2 Division Concrete	Pictorial	Abstract
Concrete	Represent the objects shared into equal parts using a bar model. 20 shared into 5 equal parts. There are 4 in each part.	Abstract Use a bar model to support understanding of the division. 18

Grouping Understand how to make equal groups from a Understand the relationship between grouping Understand how to relate division by grouping to equally whole and the division statements. repeated subtraction. $12 \div 3 = 4$ $12 \div 4 = 3$ 8 divided into 4 equal groups. There are 2 in each aroup. q 10 There are 4 groups now. $12 \div 6 = 2$ 12 divided into groups of 3. $12 \div 3 = 4$ $12 \div 2 = 6$ There are 4 aroups. Using known Understand the relationship between Link equal grouping with repeated subtraction Relate times-table knowledge directly to division. times-tables to multiplication facts and division. and known times-table facts to support division. $1 \times 10 = 10$ solve divisions $2 \times 10 = 20$ Lused the 10 $3 \times 10 = 30$ times-table $4 \times 10 = 40$ to help me. $5 \times 10 = 50$ $3 \times 10 = 30$. $6 \times 10 = 60$ 40 divided by 4 is 10. $7 \times 10 = 70$ $8 \times 10 = 80$ Use a bar model to support understanding of I know that 3 groups of 10 makes 30, so I know the link between times-table knowledge and that 30 divided by 10 is 3. division. 4 groups of 5 cars is 20 cars in total. 20 divided by 4 is 5. $3 \times 10 = 30$ so $30 \div 10 = 3$ 60 10 10